

upper Parts, under which the Colours have appear'd. I have taken notice of this so very often, that I can hardly look upon it to be accidental, and if it should prove true in general, it will bring the disquisition into a narrow compass; for it will shew that this Effect depends upon some Property, which the Drops retain, whilst they are in the upper part of the Air, but lose as they come lower, and are more mix'd with one another.

Petworth, Oct. 13.

1722.

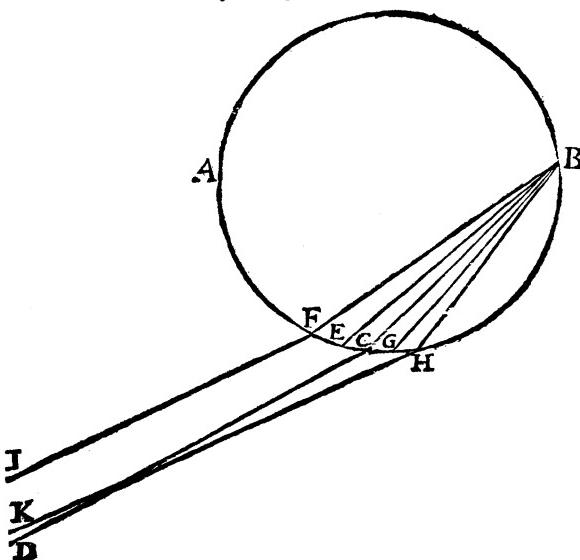
V. *A Letter to Dr. Jurin, Coll. Med. Lond. Soc. & Secr. R. S. concerning the abovementioned Appearance in the Rainbow, with some other Reflections on the same Subject. By Henry Pemberton, M. D. R. S. S.*

SIR,

UPON your communicating to me the curious Observations, your Friend Dr. *Langwith* had made on the Rainbow, I inform'd you those Appearances might, I thought, be explain'd by the Discoveries, the Great Sir *Isaac Newton* had made in the Subject of Light and Colours, in his wonderful Treatise of *Optics*. As you seemed not displeased with what I mentioned to you in relation to this Matter by word of mouth, you desired that I would set down in writing my Thoughts thereupon, which I have here accordingly done in the following manner.

R

Let



Let AB represent a Drop of Rain, B the Point from whence the Rays of any determinate Species being reflected to C, and afterwards emerging in the Line CD, do proceed to the Eye, and cause the Appearance of that Colour in the Rainbow, which appertains to this Species. It is observed by Sir Isaac Newton ^a, that in the Reflection of Light, besides what is reflected regularly, some small part of it is irregularly scattered every way. So that from the Point B, besides the Rays that are regularly reflected from B to C, some scattered Rays will return in other Lines, as in BE, BF, BG, BH, on each Side the Line BC. Further it must be noted from Sir Isaac Newton ^b, that the Rays of Light in their Passage from one Superficies of a refracting Medium to the other undergo alternate

^a Optics, Book II. Part 4.

^b Ibid. Part III. Prop. xij.

Fits of easy Transmission and Reflection, succeeding each other at equal Intervals ; insomuch that if they reach the further Superficies in one sort of those Fits, they shall be transmitted ; if in the other kind of them, they shall rather be reflected back. Whence the Rays that proceed from B to C, and emerge in the Line CD, being in a Fit of easy Transmission, the scattered Rays that fall at a small Distance without these on either side, (suppose the Rays, that pass in the Lines B E, B G) shall fall on the Surface in a Fit of easy Reflection, and shall not emerge ; but the scattered Rays, that pass at some Distance without these last, shall arrive at the Surface of the Drop in a fit of easy Transmission, and break through that Surface Suppose these Rays to pass in the Lines B F, B H ; the former of which Rays shall have had one Fit more of easy Transmission, and the latter one Fit less, than the Rays that pass from B to C. Now both these Rays, when they go out of the Drop, will proceed by the Refraction of the Water in the Lines F I, H K, that will be inclined almost equally to the Rays incident on the Drop, that come from the Sun, but the Angles of their Inclination will be less than the Angle, in which the Rays emerging in the Line CD are inclined to those incident Rays. And after the same manner Rays scattered from the Point B, at a certain Distance without these, will emerge out of the Drop, while the intermediate Rays are intercepted ; and these emergent Rays will be inclined to the Rays incident on the Drop in Angles still less than the Angles, in which the Rays F I and H K are inclined to them ; and without these Rays will emerge other Rays, that shall be inclined to the incident Rays in Angles yet less. Now by this means will be formed of every kind of Rays, besides the principal Arch which goes to the Formation

tion of the Rainbow, other Arches, within every one of the principal, of the same Colour, though much more faint : and this for divers Successions, as long as these weak Lights, which in every Arch grow more and more obscure, shall continue visible. Now as the Arches produced by each Colour will be variously mixed together, the diversity of Colours observed by Dr. *Langwith* may well arise from them.

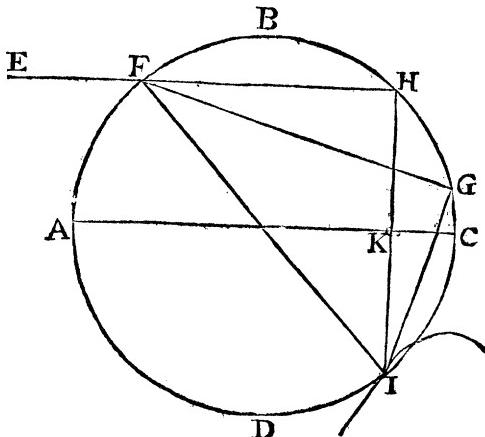
The precise Distances between the principal Arch of each respective Colour and these fainter correspondent Arches depend on the Magnitude of the Drops of Rain. In particular, the smallest Drops will make the secondary Arches of each Species at the greatest Distance from their respective principal, and from each other. Whence, as the Drops of Rain increase in falling, these Arches near the Horizon by their great Nearness to their respective principal Arches become invisible.

AND now, Sir, we are upon the Rainbow, I shall here take the Freedom of setting down two Propositions, which I have formerly considered, relating to this Subject. For the greater Brevity I shall deliver them under the Form of Porisms ; as, in my Opinion, the Ancients called all Propositions treated by Analysis only.

P R O P O S I T I O N I.

In a given refracting Circle, whose refracting Power is given, the Ray is given in Position, which passing parallel to a given Diameter of the Circle is refracted by that Circle to a Point given in the Circumference of it.

Let $A B C D$ be the given Circle, the given Diameter $A C$, and given Point G ; and let the Ray $E F$, parallel to $A C$, be refracted to G . I say $E F$ is given in Position.

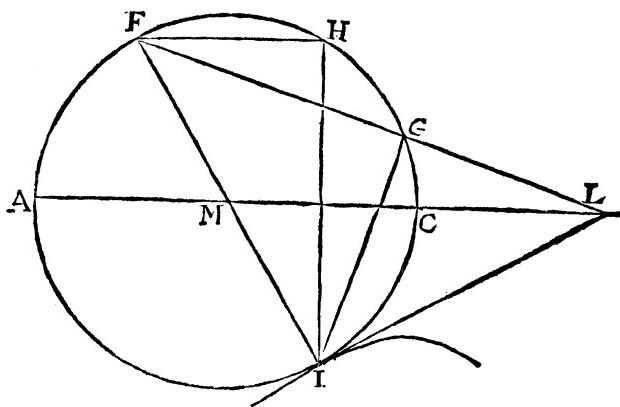


Produce $E F$ to H , and draw the Diameter $F I$, drawing likewise $I K H$, $I G$. Then is $H F I$ the Angle of Incidence, and $G F I$ the refracted Angle; so that $I H$ being perpendicular to $F H$ and $I G$ perpendicular to $F G$, $I H$ is to $I G$ as the Sine of the Angle of Incidence to the Sine of the refracted Angle, and the *Ratio* of $I H$ to $I G$ is given, as likewise the *Ratio* of $I K$ to $I G$. Therefore $I K$ being perpendicular to $A C$ the Point I is in a Conic Section given in Position, whose Axis is perpendicular to $A C$, and one of its *Foci* is the Point G ^a. Consequently the Points I and F are given, and lastly the Ray $E F$ given in Position.

^a See Papp. l. 7. prop. 238. Milnes Conic. part. 4. prop. 9.

DETERMINATION.

IT is evident, that this conic Section, may either cut the Circle in two Points, touch it in one Point, or fall wholly without it. Therefore let the Section touch the Circle in the Point I, and let IL touch both the Section and the Circle in the same point I. Then GL being joined, the Angle under $\angle IGL$ on ac-



count of the conic Section is a right one^a, so that FGL is one continued right Line, and IF is to IL as FG to GI ; as likewise, M being the Center of the Circle, MI to IL , or FH to HI , as FG to twice GI , because MI is to IF as GI to twice GI . Hence by Permutation FH is to FG as HI to twice GI ; that is, as the Sine of the Angle of Incidence to twice the Sine of the refracted Angle.

^a De la Hire Conic. lib. 8. prop. 23.

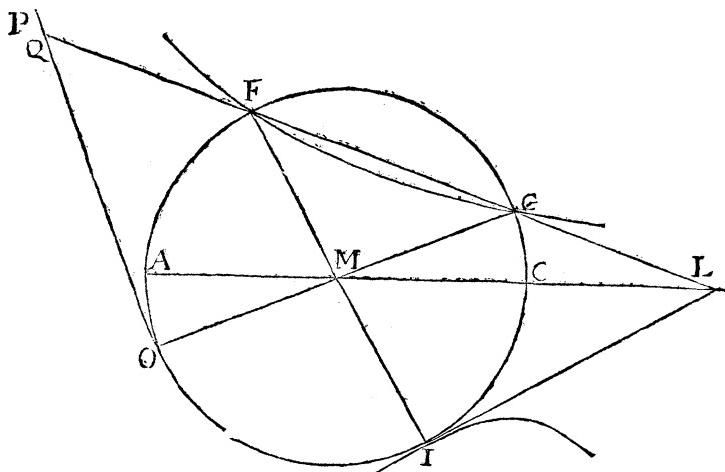
Moreover

Moreover F H being to H I as F G to twice G I, the Square of F H will be to the Square of H I, as the Square of F G to four times the Square of G I. Therefore, by Composition, as the Square of F H to the Square of F I or of A C, so is the Square of F G to the Square of F I together with three times the Square of G I, and so likewise is the Excess of the Square of F G above the Square of F H, which equals the Excess of the Square of I H above the Square of I G, to three times the Square of G I; for as one Antecedent to one Consequent, so is the difference of the Antecedents to the difference of the Consequents. Hence in the last place, the Square of half F H will be to the Square of A M, as the Excess of the Square of I H above the Square of I G to three times the Square of I G, or as the Excess of the Square of the Sine of Incidence above the Square of the Sine of Refraction, to three times the Square of the Sine of Refraction.

Another

Another DETERMINATION.

Draw the Diameter GO and the Tangent OP , meeting GF produced in Q : then the Angle under IFG is equal to the Angle under OGF , the Angle



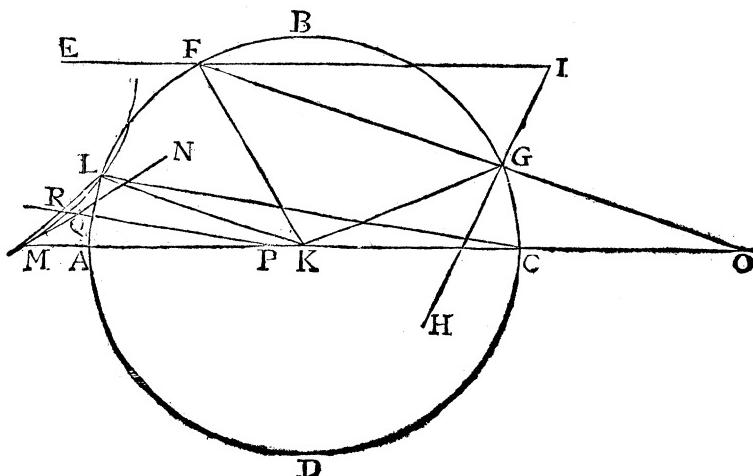
under FIL equal to that under GOQ , both being right, and FI is equal to GO ; whence the Triangles GOQ , FIL are similar and equal; so that GQ is equal to FL , and the Point F in an *Hyperbola* passing through G , whose Asymptotes are AC and OP ^a.

^a Apoll. Conic. l. 2, prop. 8.

P R O P O S I T I O N II.

A refracting Circle and its refracting Power being given, the Ray is given in Position, which, passing parallel to a given Diameter of the Circle, after its Refraction, is so reflected from the farther Surface of the Circle, as to be inclined to its incident Course in a given Angle.

Let A B C D be the given Circle ; let A C be the given Diameter, E F the incident Ray parallel to it, which being refracted into the Line F G shall so be reflected from the Point G in the Line G H, that E F and H G being produced, till they meet in I, the Angle under E I H shall be given.



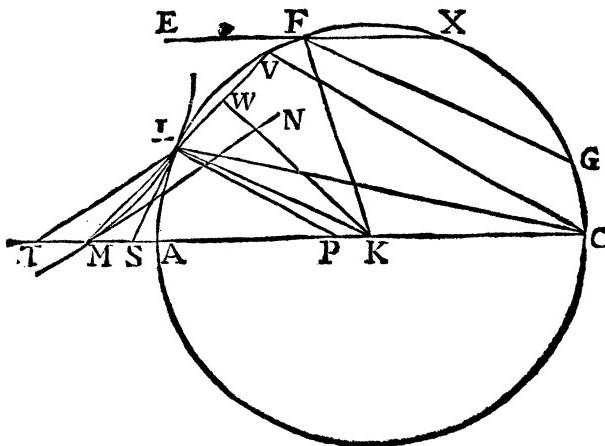
Let K be the Center of the Circle, and K F, K G be joined ; let the Semidiameter L K be parallel to the S refracted

refracted Ray FG, and MK being taken to the Semi-diameter of the Circle in the *Ratio* of the Sine of Incidence to the Sine of Refraction; let LM be joined, and lastly make the Angle under KMN equal to half the given Angle under EIH. This being done, if FG be produced to O, FO shall be to KO as the Sine of the Angle of Incidence to the Sine of the refracted Angle, that is as MK to KL; in so much that KL being parallel to FO, and the Angle under MKL equal to that under FOK, the Angle under MLK shall be equal to that under FKO, and the Angle under KML equal to that under KFO equal to that under FGK or half that under FGH, whence the Angle under KMN being equal to half the Angle under FIH, the residuary Angle under NML will be equal to half the Angle under IFG or to half that under MKL. Therefore LC being drawn, the Angle under LMN will be equal to that under MCL; and in the last place, if MC be divided into two equal Parts in P, and PQR be drawn parallel to CL, the Angle under QMR will be equal to that under RPM, and the Triangles QMR, MPR similar, so that the Rectangle under PRQ shall be equal to the Square of MR. Whence RL being equal to MR, the Point L shall be in an equilateral *Hyperbola*, touching the Line MN in the Point M, and having the Point P for its Center^a. But this *Hyperbola* is given in Position, and consequently the Point L, the Angle under MLK, and the equal Angle under CKF will be given, and therefore the Ray EF is given in Position.

^a Apoll. Conic. lib. 1. prop. 37. compared with lib. 7. prop. 23.

DETERMINATION.

Let the *Hyperbola* touch the Circle in the Point L, and let their common Tangent be LS; draw LT parallel to MN, so as to be ordinatorily applied in the *Hyperbola* to the Diameter CM. Whence LS touching the *Hyperbola* in L, PT will be to TL as TL to TS^a, and the Angle under TSL equal to that under TLP, but as the Angle under SCL is equal to that under NML, the same is equal to the Angle under TLM; therefore the Angle under SLC is equal to the Angle under MLP. Farther, ML being produced to V and VC joined, the Angle under LVC is



equal to that under SLC, by reason that LS touches the Circle in L; hence the Angles under LVC and under MLP are equal, LP, VC are parallel, and

^a Apoll. Conic. lib. 1. prop. 37. compared with lib. 7. prop. 23.

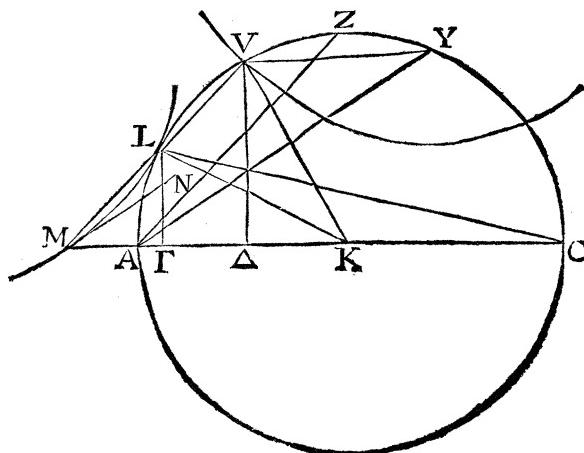
M P being equal to P C, M L is equal to L V ; and K W being let fall perpendicular to LV, M W is equal to three times L W. But now if the incident Ray E F be produced to X, the Angle under M L K being equal to that under C K F, or to that under E F K, F X shall be equal to LV, equal to twice L W ; and the Angle under K M L being equal to that under K F G ; since K W is perpendicular to M W, F G shall be to twice M W as M K to K F, or as the Sine of Incidence to the Sine of Refraction : whence M W being equal to three times L W, F X shall be to F G as the Sine of Incidence to three times the Sine of Refraction.

Moreover, M W being equal to three times L W, the Square of M W will be equal to nine times the Square of L W, and the Rectangle under V M L, or the Rectangle under C M A, that is, the Excess of the Square of K M above the Square of K A, will be equal to eight times the Square of L W ; therefore the Square of L W or the Square of half F X will be to the Square of K L, or of K A, as the Excess of the Square of K M above the Square of K A to eight times the Square of K A, that is, as the Excess of the Square of the Sine of Incidence, above the Sine of Refraction to eight times the Square of the Sine of Refraction.

Another

Another DETERMINATION.

Draw A Y parallel to M N, and A Z parallel to M V : then is the Angle under $YA\dot{Z}$, equal to that under $L M N$, which is equal to that under $L C A$; whence the Arches $A L$, YZ are equal ; but the



Arches A L, V Z are likewise equal, because LV, AZ are parallel, therefore Y V being joined, and L F drawn perpendicular to A C, the Chord V Y shall be the double of L F ; but V Δ being likewise let fall perpendicular to A C, because M V is the double of M L, V Δ shall be the double of L F ; and therefore V Δ and V Y shall be equal ; whence the point V shall be in a *Parabola*, whose *Focus* is the Point Y, its *Axis* perpendicular to A C, and the *Latus rectum*, belonging to that Axis, equal to twice the perpendicular let fall from Y upon A C ². But if K V be joined, the

² Vide de la Hire Sect. Conic. lib. 8. prop. 1, 3.

Angle

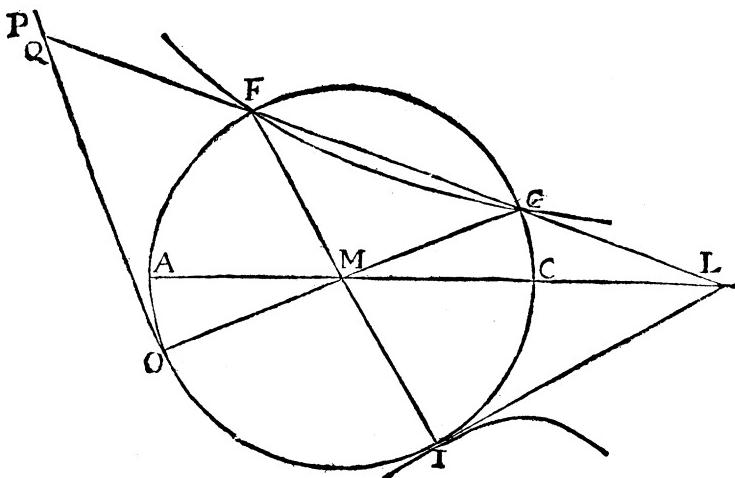
Angle under L K V is equal to twice the Complement to a right Angle of the Angle under K L V, which is equal to the Angle of Incidence, and exceeds the refracted Angle by the Angle under A K L.

T H E Determinations of these two Propositions, have relation to the first and second Rainbow ; those of the first Proposition respecting the interior, and those of the second the exterior. The first Determinations of these two Propositions assign the Angles, under which each Rainbow will appear in any given refracting Power of the transparent Substance, by which they are produced ; the latter Determinations of these Propositions teach how to find the refracting Power of the Substance, from the Angles under which the Rainbows appear ; the Angle under C M G, in the Determinations of the first Proposition, being half the Angle which measures the Distance of the interior Bow from the Point opposite to the Sun ; and in the Determinations of the second Proposition, the Angle under C M N is half the Complement to a right Angle of half the Angle that measures the Distance of the exterior Bow, from the Point opposite to the Sun. But whereas these latter Determinations require solid Geometry, it may not be amiss here to shew how they may be reduced to Calculation, seeing the Observation of these Angles, as the learned Dr. *Halley* has already remark'd ^a, affords no inconvenient Method of finding the refracting Power of any Fluid, or indeed of any transparent Substance, if it be formed into a spherical

^a *Philosoph. Transact.* No. 267. pag. 722.

or

or cylindrical Figure. For this purpose therefore I

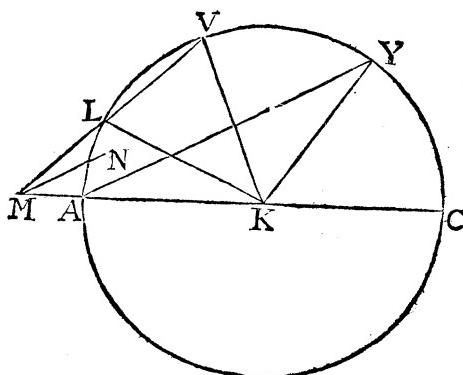


have found, that in the latter Determination of the first Proposition, if the Sine of the Angle under CMG be denoted by a , the Tangent of the Complement of this Angle to a right one be denoted by b , and the Secant of this Complement by c ; the Root of this Equation $z^3 - 3aa z = 2aa \times 2c - a$ will exceed the Sine of the Angle under FMA, that is the Sine of the Angle of Incidence, by the Sine of the Angle under CMG; and the Sine of the Angle under FMO, which is double the refracted Angle, will be the Root of this Equation $x^3 + 3aa x = 4ab$; this Angle being acute, when the Tangent of the Angle under CMG is less than half the Radius, or when the Angle itself is less than 26 degr. 33'. 54''. 11'''', and when this Tangent is more than half the Radius, the Angle under OMF is obtuse.

The

The Roots of these cubic Equations are found by seeking the first of two mean Proportionals, between each of the versed Sines appertaining to the Arches C G, A G, and the Sine of those Arches, counting from the versed Sines ; for the Sum of these two mean Proportionals is the Root of the former Equation, and the difference between them the Root of the latter ; as may be collected from *Cardan's Rules*.

And hence likewise if the first and last of the five mean Proportionals, between the Sine and Cosine of half the Angle under C M G be found, twice the Sum of the Squares of these mean Proportionals applied to the *Radius* exceeds the Sine of the Angle of Incidence by the Sine of the Angle under C M G ; and twice the difference of the Squares of the same mean Proportionals applied to the *Radius* is equal to the Sine of double the refracted Angle. Moreover this double of the refracted Angle exceeds the Angle of Incidence by the Angle under C M G.



In the latter Determination of the second Proposition draw K Y, and A Y being parallel to M N, the Angle under C K Y will be equal to twice the Angle under

under CMN, that is equal to the Complement of half the Distance of the exterior Rainbow from the Point opposite to the Sun. Then putting a for the *Radius AK*, and b for the Sine of the Angle under CKY, the Sine of the Angle under AKV will be the Root of this Equation $y^4 + 4by^3 - 8aab y + 4aab b = 0$. But the Angle of Incidence and Refraction may also be found as follows.

Let two mean Proportionals between the *Radius* and the Sine of the Angle under CKY be found, then take the Angle, whose Cosine is the first of these mean Proportionals, counting from the *Radius*; and also the Angle, whose Sine together with the second mean Proportional shall be to the *Radius* as the Cosine of the Angle under CKY to the Sine of the Angle before found. The Sum of these three Angles is double the Complement to a right one of the Angle under AKL, the Angle under KML, or the refracted Angle, being equal to half the Sum of this Angle under AKL and the Angle under CKY; as in the last Place the Angle under KLV, that is the Angle of Incidence, equal to the Sum of the Angles under KML and under MKL.

I need not observe, that the geometrical Methods of deducing these Angles of Incidence and Refraction from the Angle measuring the Distance of each Rainbow from the Point opposite to the Sun, afford very expeditious mechanical Constructions.